ME 243

Mechanics of Solids

Lecture 3: Thin walled and thick walled pressure vessels

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Thin walled pressure vessels

- For a thin walled pressure vessel, the diameter (D) is at least 20 times greater than its thickness (t).

i.e. $D \geq 20t$

Or, $t \leq \frac{D}{20}$

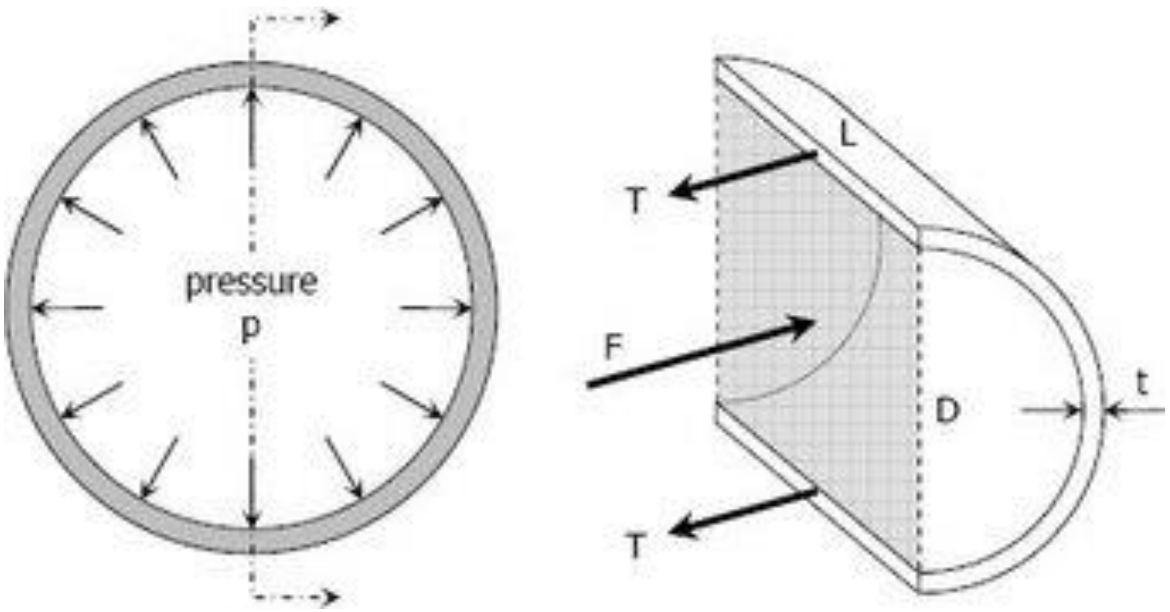
Thin walled pressure vessels

- A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.
- Two types of stresses are developed:

1. Tangential/ circumferential/ hoop stress.
2. Longitudinal stress

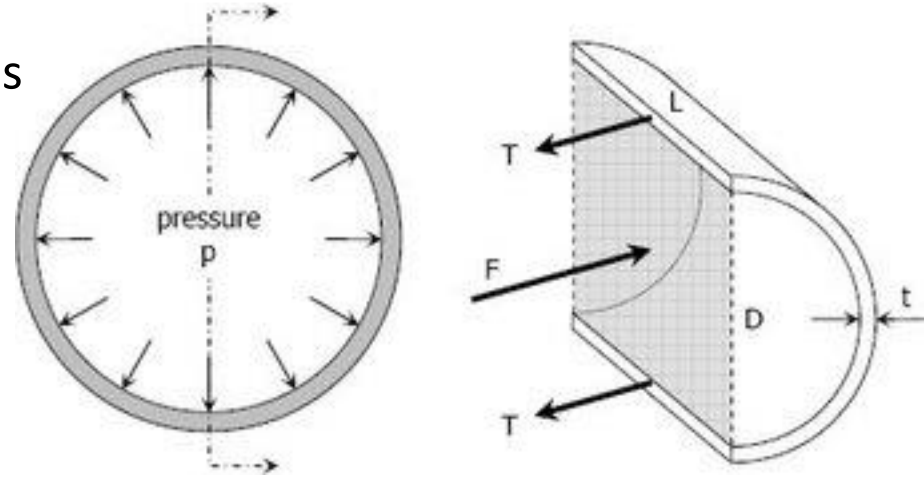
Tangential stress

- Consider the tank shown being subjected to an internal pressure p. The length of the tank is L and the wall thickness is t. Isolating the right half of the tank:



Tangential stress(cont...)

The forces acting are the total pressures caused by the internal pressure p and the total tension in the walls T.



$F = pA = pDL$

$T = \sigma_t A_{wall} = \sigma_t tL$

Now, $\Sigma F_H = 0$

or, $F = 2T$

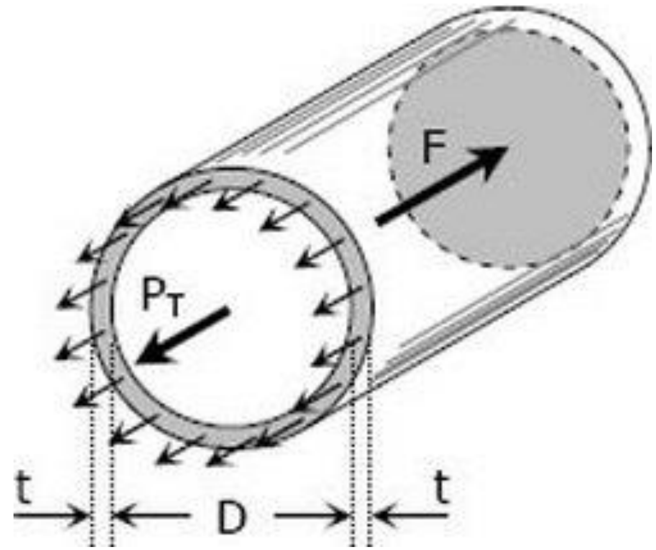
Or, $pDL = 2(\sigma_t tL)$

Therefore,

$$\sigma_t = \frac{pD}{2t}$$

Longitudinal stress

- Consider the free body diagram in the transverse section of the tank:



- The total force acting at the rear of the tank F must equal to the total longitudinal force on the wall,

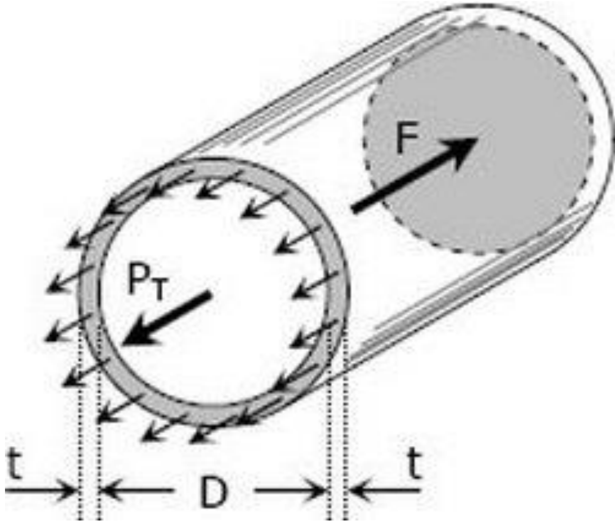
  $P_T = \sigma_L A_{wall}$.

- Since t is so small compared to D, the area of the wall is close to πDt

Longitudinal stress (cont..)

$$F = pA = p\frac{\pi}{4}D^2$$

$$P_T = \sigma_L \pi D t$$



$$\Sigma F_H = 0$$

$$P_T = F$$

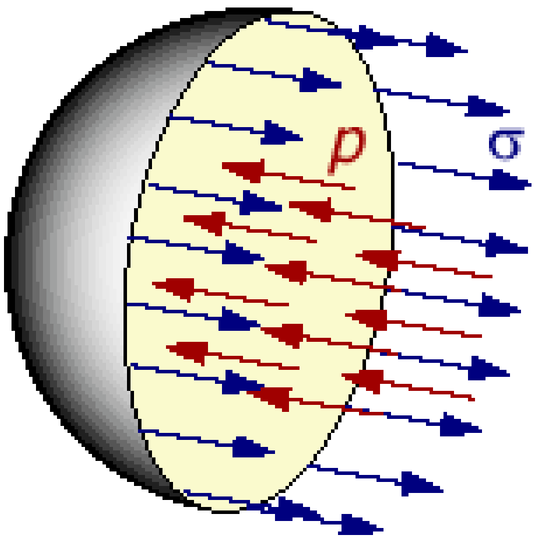
$$\sigma_L \pi D t = p\frac{\pi}{4}D^2$$

Therefore, $$\sigma_L = \frac{pD}{4t}$$

Spherical pressure vessel

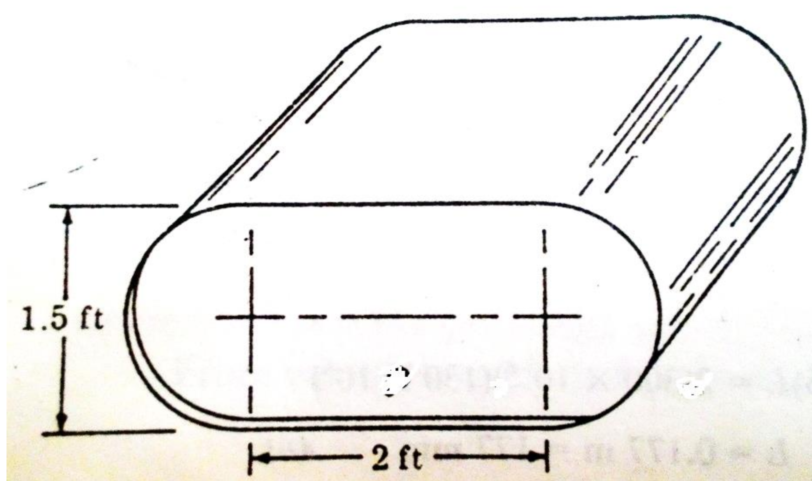
- Always longitudinal stress (σ_L) develops in a spherical pressure vessel, which is given by,

$$\sigma_L = \frac{pD}{4t}$$



Problem

- The tank shown in Fig. is fabricated from 1/8-in steel plate. Calculate the maximum longitudinal and circumferential stress caused by an internal pressure of 125 psi.

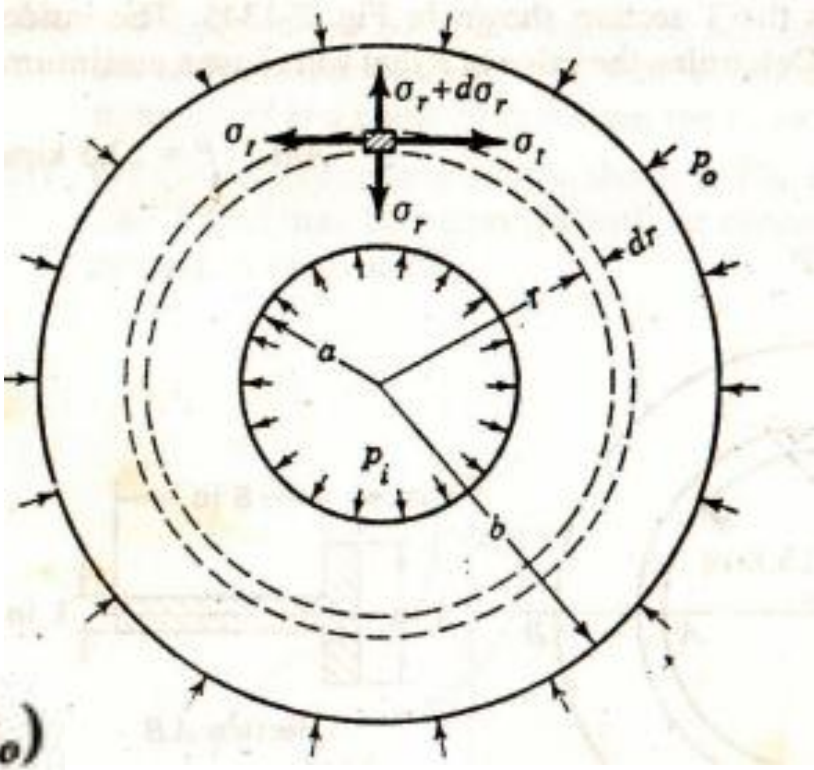


Thick walled pressure vessel

- A pressure vessel is considered as thick walled, if $t > \frac{D}{20}$
- Radial stress is considered along with hoop stress and longitudinal stress.
- Stresses at any point on a thick walled cylinder in terms of applied pressures and dimensions are first determined by Gabriel Lame in 1833.

Thick walled pressure vessel

The cylinder shown in figure has radii ‘a’ and ‘b’ and is subjected to both a uniformly distributed internal pressure ‘p_i’ and external pressure ‘p_o’.



Solving for σ_r and σ_t we get,

$$\sigma_r = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} - \frac{a^2 b^2 (p_i - p_o)}{(b^2 - a^2) r^2}$$

$$\sigma_t = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{a^2 b^2 (p_i - p_o)}{(b^2 - a^2) r^2}$$

Thick walled pressure vessel: special cases

- Case 1: Internal pressure only:

Here, the external pressure is zero. So we get,

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

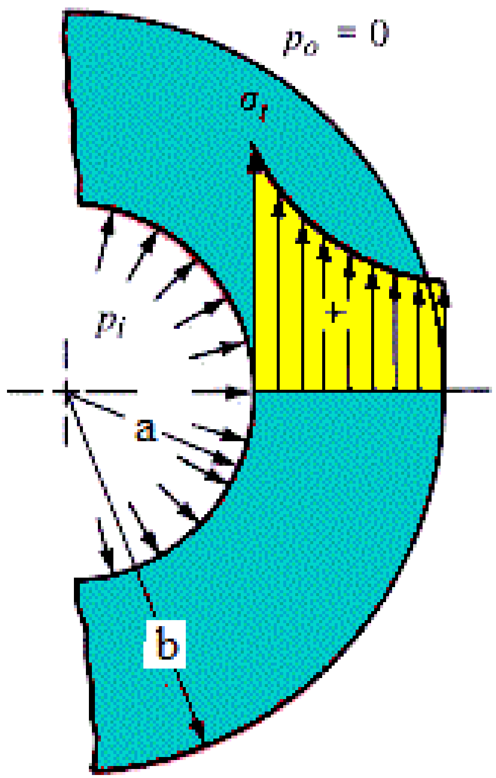
$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

Here,

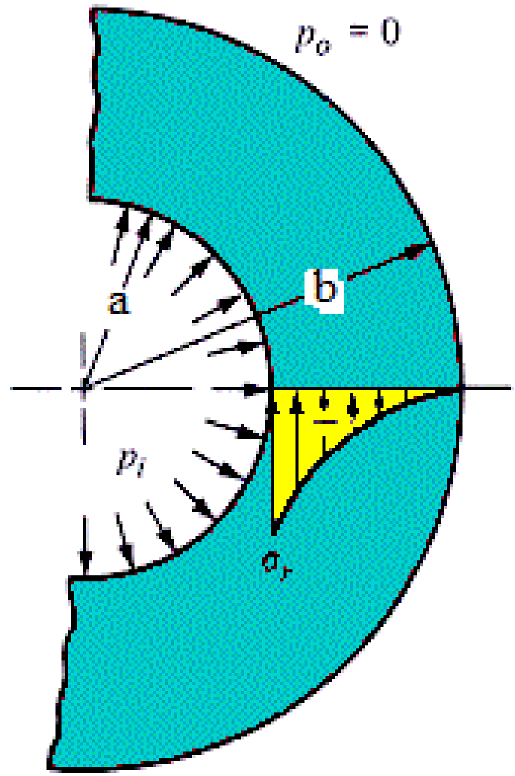
- σ_r is always compressive
- σ_t is always tensile
- σ_t is larger than σ_r and is maximum at inside surface of the cylinder, and is given by,

$$(\sigma_t)_{max.} = \left(\frac{b^2 + a^2}{b^2 - a^2}\right)p_i$$

Thick walled pressure vessel: special cases (cont…)



(*a*) Tangential stress distribution



(*b*) Radial stress distribution

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

Thick walled pressure vessel: special cases (cont…)

- Maximum shear stress is found at inner surface of the cylinder where σ_t and σ_r are maximum and is given by,

$$\tau_{max.} = \frac{(\sigma_t)_{max.} - (\sigma_r)_{max.}}{2} = \frac{b^2}{b^2 - a^2} p_i$$

Thick walled pressure vessel: special cases

- Case 2: external pressure only:

Here the internal pressure is zero. So we get,

$$\sigma_r = -\frac{p_o b^2}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right)$$

$$\sigma_t = -\frac{p_o b^2}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right)$$

Here,

- Both σ_r and σ_t are always compressive.
- σ_t is larger than σ_r and is maximum at inside surface of the cylinder, and is given by,

$$(\sigma_t)_{max.} = -\frac{2b^2 p_o}{b^2 - a^2}$$

Assignment: 2

- Derive the expressions of tangential stress and radial stress for a thick wall pressure vessel and solve for special cases. Also show the distribution of stresses for each case.
- Solve Prob. 1346 of singer's book.